Lambda Calculus

For the following forms apply β-reduction and α-substitution to reduce to lowest form. Indicate at each step the rule you are applying.

1. (λx.x)(λx.x)

* (λx.x)(λx.x) 🡪 (λx.x)(λy.y) [α-substitution]
* (λx.x)(λy.y) 🡪(λy.y) [β-reduction] [Identity]

1. (λx.x x) (λx. λy.x x)

* (λx.x x) (λx. λy.x x) 🡪 (λx.x x) (λt. λy.t t) [α-substitution]
* (λx.x x) (λt. λy.t t) 🡪 (λt. λy.t t) (λt. λy.t t) [β-reduction]
* (λt. λy.t t) (λt. λy.t t) 🡪 λy.(λt. λy.t t) (λt. λy.t t)
* No final form available – reduction never ends

1. ((λx.(x y))(λz.z))

* ((λx.(x y))(λz.z)) 🡪 (λz.z y) [β-reduction]
* (λz.z) y) 🡪 y

1. (λz.z) (λy.y y) (λx.x a)

* (λz.z) (λy.y y) (λx.x a) 🡪 (λy.y y) (λx.x a) [β-reduction]
* (λy.y y) (λx.x a) 🡪 (λx.x a) (λx.x a) [β-reduction]
* (λx.x a) (λx.x a) 🡪 (λx.x a) a [β-reduction]
* (λx.x a) a 🡪 a a [β-reduction]

1. (λz.z) (λz.z z) (λz.z y)

* (λz.z) (λz.z z) (λz.z y) 🡪 (λz.z) (λt.t t) (λz.z y) [α-substitution]
* (λz.z) (λt.t t) (λz.z y) 🡪 (λz.z) (λt.t t) (λs.s y) [α-substitution]
* (λz.z) (λt.t t) (λs.s y) 🡪 (λt.t t) (λs.s y) [β-reduction]
* (λt.t t) (λs.s y) 🡪 (λs.s y) (λs.s y) [β-reduction]
* (λs.s y) (λs.s y) 🡪 (λs.s y) y [β-reduction]
* (λs.s y) y 🡪 y y [β-reduction]

1. (λx.λy.x y y) (λa.a) b

* (λx.λy.x y y) (λa.a) b 🡪 (λx.(λy.x y y)) (λa.a) b
* (λx.(λy.x y y)) (λa.a) b 🡪 (λy.(λa.a) y y) b [β-reduction]
* (λy.(λa.a) y y) b 🡪 (λy.y y) b [β-reduction]
* (λy.y y) b 🡪 b b [β-reduction]

1. (λx.x x) (λy.y x) z

* (λx.x x) (λy.y x) z 🡪 (λt.t t) (λy.y x) z [α-substitution]
* (λt.t t) (λy.y x) z 🡪 (λy.y x)(λy.y x) z [β-reduction]
* (λy.y x)(λy.y x) z 🡪 ((λy.y x) x) z [β-reduction]
* ((λy.y x) x) z 🡪 x x z [β-reduction]

1. (λx. (λy. (x y)) y) z

* (λx. (λy. (x y)) y) z 🡪 (λx. (λt. (x t)) y) z [α-substitution]
* (λx. (λt. (x t)) y) z 🡪 (λx. (x y)) z [β-reduction]
* (λx. (x y)) z 🡪 z y [β-reduction]

1. ((λx.x x) (λy.y)) (λy.y)

* ((λx.x x) (λy.y)) (λy.y) 🡪 ((λx.x x) (λy.y)) (λt.t) [α-substitution]
* ((λx.x x) (λy.y)) (λt.t) 🡪 ((λy.y)(λy.y)) (λt.t) [β-reduction]
* ((λy.y)(λy.y)) (λt.t) 🡪 (λy.y) (λt.t) [β-reduction]
* (λy.y) (λt.t) 🡪 (λt.t) [β-reduction] [Identity]

1. (((λx. λy.(x y))(λy.y)) w)

* ((((λx. λy.(x y))(λy.y)) w) 🡪 ((((λx. λy.(x y))(λt.t)) w) [α-substitution]
* ((((λx. λy.(x y))(λt.t)) w) 🡪 (((λy.( λt.t) y) w) [β-reduction]
* (((λy.(λt.t) y) w) 🡪 (λt.t) w [β-reduction]
* (λt.t) w 🡪 w [β-reduction] [Identity]